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Carry Trade and Return Crash Risk*

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Abstract

We build two leveraged and non-leveraged strategies for carry trading. In the non-leveraged carry trade we show that the Sharpe ratio as a proxy for profitability has a concave form with respect to the interest rate differentials. Our model predicts the concavity of the Sharpe ratio and data confirm it as well. However, high interest rate currencies have greater currency crash risk exposure and this is investigated through skewness and kurtosis of 9 most used currencies against Japanese yen. Skewness has a decreasing relationship and kurtosis an increasing relative to the interest rate differentials. We also simulate the positive shocks to the interest rates and negative shock to exchange rates skewness and we study the response of profit. Finally for the leveraged strategy, we show whenever the volatility of the funding currency is low and its covariance with the target currency is large enough, the trader should increase its short-selling position.

Keywords: carry trade, crash risk, leverage risk premium, Sharpe ratio

JEL Classification: E44, F31, G12

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1 Introduction

A currency carry trade is defined as a leveraged cross-currency position to take advantage from interest rate differentials and high Sharpe ratio of this market. The strategy consists of borrowing fund from banks in a low interest rate currency and investing it in a currency with higher interest rate. The former currency is often called funding currency and the latter one target currency. Notice that the strategy is profitable for an unhedged carry trade strategy, when the interest rate differential is high enough to compensate exchange rate fluctuations and so the uncovered interest rate parity (UIP) is not expected to hold. The UIP is a condition of no-arbitrage in which the currency with higher return (interest rate) should depreciate against the currency with lower return. Therefore under UIP the profit through interest rate differentials is offset by the exchange rate movements. In fact traders enter into the carry trade market whenever they think the UIP will not hold. That means the exchange rate movements will not necessarily offset those differences between countries' interest rates. Thus violation of the uncovered interest parity is probably the most interesting feature of carry trade for speculators who choose the leveraged position through this expectation.

Although most of the time carry trade strategies consist of investing in short-term periods, the UIP holds only in long-term period. Chinn and Meredith (2006) confirm that the UIP holds for periods longer than 5 years. In fact what happens in reality is inverse of what the UIP predicts. Many papers discussed that currencies with high interest rates tend to appreciate while other currencies with low interest rates depreciate. This is called the Forward Premium Puzzle. Furthermore some researchers suggest that forward premium puzzle is not necessarily a pre-condition to engage in the carry trade market, but it can be the consequence of carry trade. The idea is introduced by Froot and Thaler (1990). They tested unbiasedness of exchange rate changes provided by interest rate differential with this equation:

$$\Delta s_{t+k} = \alpha + \beta(i_t - i_t^*) + \nu_{t+k}$$

where Δs_{t+k} is the exchange rate change over k periods, $(i_t - i_t^*)$ is the interest rate differentials and ν is the error terms. The null hypothesis is $\beta = 1$. The result is that β is less than one and indeed it is negative. The negative β means that the money which offers higher interest rate tends to appreciate. The authors thought a possible explanation could be, the slow response of some participants to interest rates differential changes. The failure of the UIP is known to market participants and therefore some of them have created tradable benchmarks and they have introduced FX instrument referencing these benchmarks. Gyntelberg and Remolona (2007) mentioned some of these indexes in their paper.

Our contribution in this paper consists of ranking currencies using the Sharpe ratio, skewness and kurtosis. We show that the Sharpe ratio as a performance measure for profitability has a concave form, theoretically and empirically. Also we show in data that skewness has a decreasing and kurtosis has an increasing relationship with interest rate differentials. In other words, high interest rate differential compensates investors for bearing a high risk of currency crash. Skewness and kurtosis are the measures for exchange rate crash risks (Brunnermeier et al (2008)). We put the stress on the Sharpe ratio in this research and therefore we assumed normality for the return process. The Sharpe ratio takes into account the first two moments and since it is based on mean-variance theory, the normality hypothesis should be imposed on the return process. Thus the Sharpe ratio can be useful only if a return risk depends on the variance of the distribution. According to the mean-variance and the CAPM theories, portfolios with highest Sharpe ratio are mean-variance efficient. Therefore, we could not show the decreasing relationship of skewness theory since the return in our model follows a normal distribution.

We present a model in which the arbitrageurs perform carry trade strategies. They borrow from a country with low interest rate and invest it in a country with high interest rate. The arbitrageurs return function has a log-normal form and they have a power utility function. Solving this system result in a mean-variance utility function. Therefore in this model every arbitrageur holds mean-variance utility function. Consequently, in equilibrium the market portfolios are mean-variance efficient. We then show the condition for which the arbitrageurs enter into the carry trade market. This condition has a close relationship with the UIP condition. However, we do not make any assumption about what is driving UIP rejections.

Like any other competitive but not perfect markets, number of agents involving in the market influences the arbitrage opportunities, if any, in the market. Our model shows when the number of arbitrageurs grows to infinity, the arbitrage opportunity vanishes and UIP holds. The limitation of lender resources ends in this result. When the number of arbitrageurs are infinity their ability to borrow is infinite small and hence their profit goes to zero. While, this result by itself does not imply UIP condition, binding this condition is equivalent to the UIP condition in our model. As a result, the excess return and the Sharpe ratio tend to zero.

In the theoretical model, the Sharpe ratio shows a concave form against the interest rate differentials in which the maximum corresponds to a mean-variance efficient portfolio. This result is confirmed by the figure 6 where the maximum happens for the Hungarian forint.

However as discussed before, Sharpe ratio is based on the first two mo-

ments. Brunnermeier et al (2008) provide some evidence that carry trade returns and involved currencies are negatively skewed. We show in the figures 2 and 3 the skewness and the kurtosis of the returns versus interest rate differential. The Bigger interest rate differentials are, the more negative is the skewness and the more positive is the kurtosis.

Finally as an exercise, we calculate the leveraged position of a trader in the spot Forex market. We deduce for example whenever the variance of funding currency is low and its covariance with the target currency is high, the trader should reinforce its short-selling position.

Carry trade does not tend to involve major currencies, instead it involves currencies with high return such as Australian dollar, New Zealand dollar, Island krona, Turkish Lira, Brazilian real, Hungarian forint and even occasionally pound sterling. Carry trade involves mostly Japanese yen and Swiss franc and sometimes US dollar as the funding currencies. However the situation of the US dollar is different since 2004. The dollar served as a potential target currency rather than a funding currency until 2007 and then with a decrease in the US interest rate it became a funding currency again.

We divide currencies into four groups. First group contains only the funding currencies which includes Swiss franc and Japanese yen. We place US dollar, UK pound and the Euro in the group of occasionally target currencies. The Third and fourth groups are the target currencies with low and high interest rate differentials respectively respect to the funding currencies. Australian and New Zealand dollar and Iceland krona are classified in the third and Brazilian, Turkish, Hungarian currencies are in the last group. Figures 7 to 10 in the appendix show the corresponding interest rate for each class of currencies.

2 Related literature

The main issue in the carry trade market is to identify the nature of risk and answer to the question whether the excess return in this market is related to this risk. This is also in close connection with the UIP rejections. However the papers differ in their assumption about the risk resources and what the principle reasons for UIP rejections are.

The first group of papers pays attention to the liquidity problem in applying carry trade strategies. Brunnermeier et al (2008) are the first authors who showed the exchange rate movements of carry trade portfolios are negatively skewed and therefore carry trades are subject to currency crash risk (and equivalently big positive return). The authors argue that this skewness in foreign exchange rates follows from temporary changes in the availability of funding liquidity to arbitrageurs. When the funding liquidity is temporarily

reduced, this results in a rapid unwinding of the traders' positions and thus leads to abrupt changes in the exchange rates, which go against the carry traders. This risk, they argue, is a major factor affecting traders' willingness to enter into these risk arbitrage positions. Brunnermeier and Pederson (2008) look at the relationship between funding liquidity and asset market liquidity, but in a general context. In their model the market liquidity can suddenly dry up. Adrian et al (2009) show also that foreign exchange markets are influenced by liquidity conditions. They used balance sheets of financial intermediaries as a tool to predict the future returns on the currency market. They found liquidity channel is related to the exchange rate risk premium and this is associated to the carry trade incentives.

The second group of papers put the stress on a state of nature in which a disaster could happen. These papers examine whether there is really a crash risk premium, however they leave the source of risk unexplained. Burnside et al (2008) refer to the peso problem as an explanation for the high average payoff to the carry trade. Peso problem is defined as a generic term for the effects of small probabilities of large events in empirical works. Their approach relies on analysing the payoffs to a version of the carry trade strategy that does not yield high negative payoffs in a peso state. This strategy works as follow. When an investor sells the foreign currency forward, he simultaneously buys a call option on that currency. If the foreign currency appreciates beyond the strike price, the investor can buy the foreign currency at the strike price and deliver the currency in fulfillment of the forward contract. Similarly, when an investor buys the foreign currency forward, he can hedge the downside risk by buying a put option on the foreign currency. By construction, this hedged carry trade does not generate large negative payoffs in the peso states. To estimate the average payoffs of the hedged carry trade, the authors used data on currency options with a one-month maturity. At this stage of the analysis Burnside et al wish to be eclectic about the size of the negative payoffs in the peso states. So, their hedging strategy uses at-the-money options which pay off in all peso states, as well as in some non-peso states. The main results of the paper are as follow: first the average payoffs to the hedged and unhedged carry trade are very similar. Second the standard deviation of the payoffs to the hedged carry trade is actually substantially lower than those of the unhedged carry trade. Third the payoffs to the unhedged carry trade in the peso states is only moderately negative and finally the SDF is over one-hundred times larger in the peso states than in the non-peso states.

Fahri et al (2009) decompose the profit in the carry trade market into the profit due to a Gaussian risk premium and the profit due to a disaster risk premium. They link the disaster risk premium to the impact of disasters on SDF and the carry trade payoffs in disaster periods. They argue that with a non-hedged carry trade, we can only compute Gaussian risk premium and

this strategy does not allow to capture disaster risk premium. In order to compute this risk premium, they use currency options. After estimation, they showed that disaster risk premium explain about 30 percent of the carry trade returns. From this point of view their strategy of hedging is very similar to those of Burnside et al. It is worth mentioning that the Peso states in the former article correspond to the disaster risk. Jurek (2008) investigates whether the excess return in the carry trade market is due to the exposure to currency crash. For this purpose, he used the dynamics of the moments of the risk-neutral distribution implied by the currency options. Also he examined the return to the carry trade market in which the risk of currency crash has been hedged by currency options (as the previous papers). The results show that the crash risk premium explains 30-40% of the total excess return in currency carry trade which is similar to the paper of Farhi et al . However taking into account the risk in the carry trade market, the excess return still remains very high. Bhansali (2007) build a hedged portfolio in the carry trade market with using the currency options. Bhansali shows that the volatility of such option is proportional to the currencies interest rate differentials. The author shows theoretical and empirical supporting evidence for positive connection between volatility and carry. Clarida et al (2009) confirms and extends this result by using two different strategies with forward contracts and currency options. They show these two risky strategies have similar payoffs and risk characteristics.

Plantin and Shin's (2008) approach is different from above mentioned papers. They use a bubble model of speculator behavior in order to explain the process of speculation in the carry trade market. They find without funding costs and with the known and constant fundamental value of asset, speculation is not possible on the market and therefore, however the presence of carry costs change completely the previous results. By introducing carry costs and allowing the fundamental value to be stochastic, they generate a speculative dynamic in this market. According to the authors, these results highlight the importance of the interaction between carry costs and the sensitivity of prices to flows. The main goal of the paper is to show that in the carry trade market, bubble can exist. They pay attention to the phenomenon of *going up by the stairs and down by the elevator* for the currencies involved in the carry trade market. Many studies on bubbles have paid attention to this phenomena. Abreu and Brunnermeier (2003) and Veldkamp (2004) report this slow boom and sudden crash in financial markets. Veldkamp explains this pattern with an endogenous flow of information that varies with the economic activity level. This information mechanism endogenously generates unconditional asymmetry in lending rate and investment changes. She introduces two measures for judging the asymmetry of data: time-irreversibility and skewness.

The spirit of our model is relatively similar to the model of Jylh et al (2008). Our result about the number of arbitrageurs and its relationship with the non-arbitrage opportunity are quite similar.

3 Data and Some Stylized facts

3.1 Some Stylized facts in Carry Trade Market

Datastream is used for the exchange rates and interest rates. The exchange rates are in daily basis for each country and we used the average for the monthly, quarterly and yearly data. We also used interest rates in the monthly, quarterly and yearly basis. In this section, we document some stylized facts in the carry trade market with a descriptive and econometric approach. The following figure is adopted from Brunnermeier et al (2008) and shows the dollar exchange rate against the yen.



Figure 1: USD/JPY exchange rate.

There were many dollar crash episodes during the shown period in the figure. However the dollar crash in October 1998 was one of the biggest. This risk (Peso state or disaster risk) can cause a big loss for traders in the carry trade market. In fact, we can decompose the profit of carry trade into the profit due to the interest rate differentials and the profit (or loss) due to the target (funding depreciation) exchange rate appreciation (depreciation). The authors found this drastic depreciation to have no relationship with fundamental news announcement but it can more be related due to the unwinding of carry trade. This is an example of *going up by the stairs and down by the elevator*. Here, we use skewness and Kurtosis as measures to calculate asymmetry of exchange rates changes.

3.2 Risk in the Carry Trade Market: A descriptive approach

In this section, Skewness and Kurtosis are used as measures of exchange rate risk. Skewness is used to show the risk of currency crash while Kurtosis measures whether these crashes are abrupt or not (We use these measures to compute the Gaussian risk in the Farhi et al paper). A big negative skewness means that the exchange rate has been appreciated slowly and is crashed suddenly while a big positive Kurtosis shows that this crash is fast. Figure (11) shows skewness and Kurtosis for some target currencies. Almost all pair of currencies have positive big Kurtosis and negative skewness. Negative skewness in this context confirms the slow boom of exchange rates and sudden crash. Big positive kurtosis confirms that these changes are fast. The most extreme case in this figure is in (11d). The measures in panel (11c) capture the high depreciation of the US dollar against the yen in October 1998. In the last quarter of 1998 the skewness of USD/JPY is -2.35 and its kurtosis attains to 10.91.

3.2.1 Currency crash risk

As discussed above, the exchange rate movements are not symmetric when it goes up and when it falls down. This is investigated through skewness and kurtosis. Skewness and kurtosis for every currency is plotted and showed in the appendix. The asymmetry of exchange rate movement is associated with a crash risk. The price of this risk (or the risk premium) is labeled as risk reversal price. Risk reversal is a long position in an out-of-the money call option combined with a short position of an equally out-of-the-money put, that is, call-put. Risk reversal is in fact an option to restrict the loss while it limits the maximum profit. Under risk neutral measure, exchange rate is symmetrically distributed and so the price of risk reversal is zero, since the value of buying a call offset the value of selling a put. But for example if the exchange rate is negatively skewed (data are skewed left) the price of risk reversal is negative.

The skewness and Kurtosis are calculated of daily exchange rate changes within each month and each quarter since July 1996. Figures (12) and (13) depict skewness and kurtosis quarterly for all 9 currencies against Japanese yen. These figures show clearly that all currencies are skewed negatively relative to Japanese yen. Panel 13j is JPY/TKL (whereas TKL/JPY is so small it was better to use JPY/TKL) exchange rate and it shows that the yen is positively skewed against Turkish lira, therefore TKL is negatively skewed. Negative skewness confirms the argument *going up by the stairs and down in the elevator*. The argument is true even for a currency like euro which is targeted less comparing to other currencies. Negative skewness is the risk for the speculators and they can assure themselves via buying risk-reversal. Almost

in all panels it can be seen that whenever skewness has a negative peak, the corresponding kurtosis has a positive peak, which means the changes occurred very rapidly. This happens especially in figure (13) which contains currencies with high interest rates.

Figure (2) shows the average skewness and the kurtosis for all currencies since July 1996 until July 2008 versus interest rate differentials. Although average skewness and average kurtosis does not show the exact situation, they are useful to understand the general tendency in the carry trade market. This figure shows where the interest rate is higher, the skewness is more negative. Also figure (3) for the average kurtosis demonstrates that higher interest rate currencies are related to higher positive kurtosis.

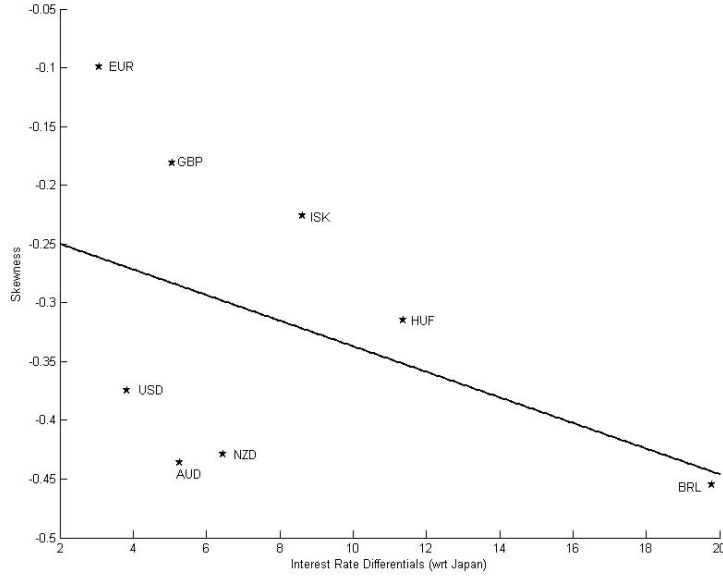


Figure 2: Average skewness vs interest rate differentials 1996-2008.

Euro has the highest skewness (in term of absolute value) and the lowest kurtosis. Also Brazilian real offers highest interest rate and it has the highest kurtosis and the lowest skewness. This pattern is true for other currencies as well. Higher is the return, more is the risk (negative skewness and positive kurtosis). Thus the figures describe well the risk associated to currency crash. According to these figures we can divide currencies into three groups: currencies with low return and low risk (euro and pound), currencies with medium return and higher risk comparing to the first group (USD, AUD, NZD) and finally the last group which contains high-return high-risk currencies (ISK, HUF, BRL). From similarity of interest rate movements, Turkey belongs to

the last group. This classification is consistent with the figures showed in the introduction for the exchange rate co-movements.

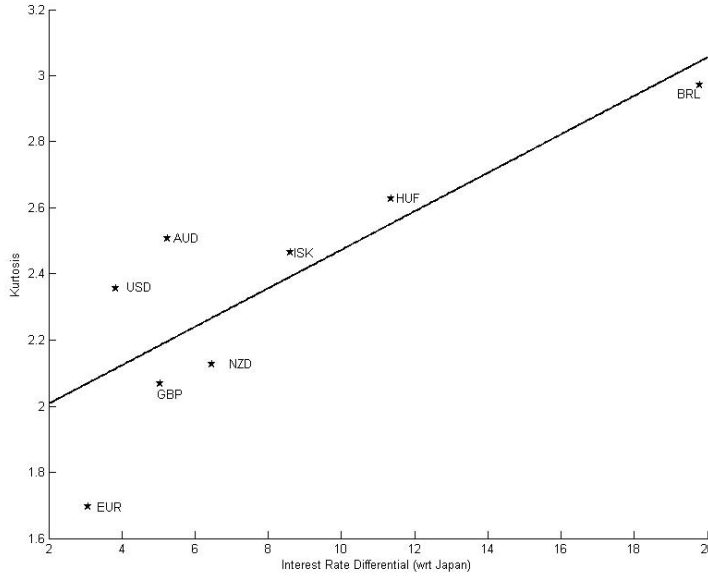


Figure 3: Average kurtosis vs IR differentials 1996-2008.

3.3 The International Common Shocks

The goal of this subsection is to identify international transmission of shocks¹. To do so, we used a Panel Vector AutoRegression (PVAR) framework. The most general form of the model can be written as:

$$\Pi_{i,t} = \mu_i + \Theta(L)\Pi_{i,t} + \epsilon_{i,t}, i = 1, \dots, N, t = 1, \dots, T$$

where $\Pi_{i,t}$ contains five variables (Skewness, Kurtosis, Exchange rate, Profit, Interest rate differential). μ_i is the country idiosyncratic effect, $\epsilon_{i,t}$ is the residual error and $\Theta(L)$ is a lag operator with $\Theta(L) = \Theta_1 L + \Theta_2 L^2 + \dots + \Theta_p L^p$. In order to compute impulse respond functions (IFR), we identify the shocks using Choleski decomposition. This decomposition introduces some restrictions on contemporaneous correlations between variables. PVAR methodology is also useful to take into account the endogeneity problem and the inter-relationship dynamic between the variables. Helmert transformation is used

¹We mainly used the STATA code developed by Inessa Love; *Financial Development and Dynamic Investment Behavior: Evidence from Panel VAR* (with Lea Ziccino), The Quarterly Review of Economics and Finance, 46(2) (May 2006), 190-210.

in order to remove the individual effects, i.e., the difference between each variable and its forward mean. The variables are sorted with the ordering Profit, Interest rate differential, Exchange rate, Skewness, Kurtosis.

A third-order PVAR have been estimated with these variables, using monthly data from July 1996 until July 2008. Some results are presented in the following figures.

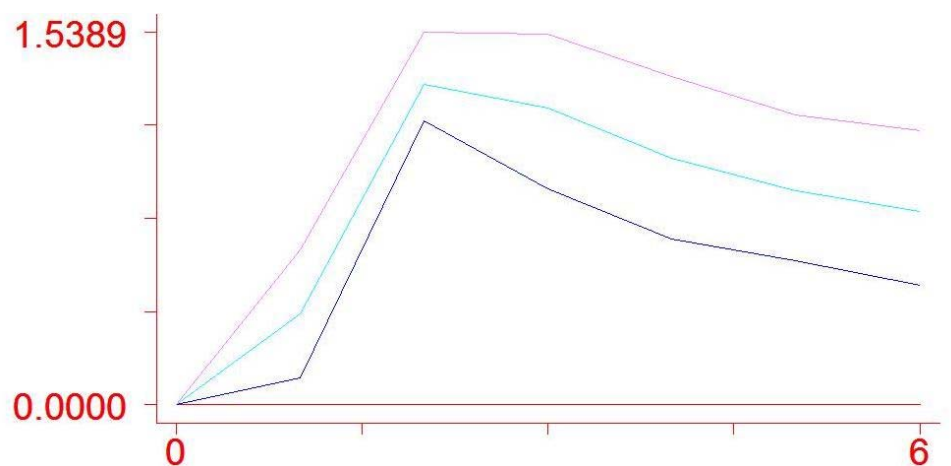


Figure 4: Response of Profit to Interest rate differentials shock

Figure (4) shows the response of profit to a positive shock to the interest rate differentials. This means that interest rate differentials have a relatively big positive impact on the payoff.

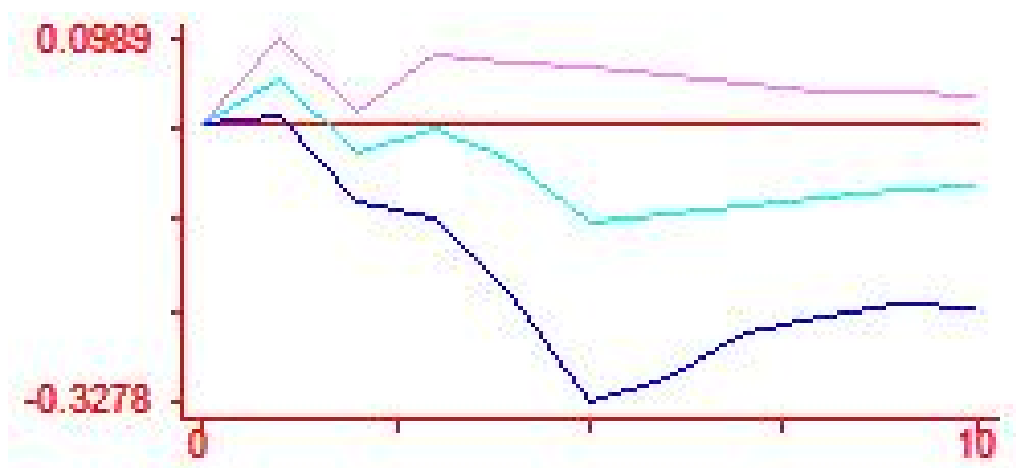


Figure 5: Response of Profit to Skewness shock

Figure (5) confirms econometrically the previous graphs which were plotted for different currencies. It shows a positive shock to skewness has a negative effect on profit. In other words, the higher the risk of currency crash is, the higher is the probability of return crash risk.

In the following sections, we will rationalize these stylized facts.

4 The Model

4.1 Setup of the model

We have three countries, the first is the supplier of the *funding currency* and the second is the investment destination *target currency* and K arbitrageurs are located in a third country. Arbitrageurs take a long position in the funding currency and a short position in the target currency. The final profit is expressed in the currency of the arbitrageur. At the end of the investment period, the arbitrageur get her money back from the target country and must repay the borrowed money from funding currency. Therefore this model has only a single period maximization program. Arbitrageurs are then exposed to two exchange rate risks: the depreciation of the target currency and/or the appreciation of the funding currency. Her profit comes from the interest rates differentials between the target and funding currency adjusted by the exchange rate movements.

For the moment, we assume the dynamic of the exogenous exchange rate as follow:

$$dS_t = S_t (\mu dt + \sigma dw_t)$$

where μ and σ are the drift and the volatility of the exchange rate and w_t is a Brownian motion. We also assume the volatility and the drift of this process are exogenous. Thus we do not care about the risk resource or what drives UIP rejections. Using Ito's lemma, it is easy to show the level of the exchange rate can be written as:

$$S_t = S_0 e^{\left(\mu - \frac{\sigma^2}{2}\right)t + \sigma(w_t)}.$$

So, $\frac{S_{t+1}}{S_t}$ follows a log-normal distribution with a mean of $\left(\mu - \frac{\sigma^2}{2}\right)$ and a variance of σ^2 .

4.2 Dynamic of wealth and Utility function

We suppose arbitrageurs have no initial wealth at time t . At time $t + 1$, the wealth of the representative arbitrageur k is:

$$W_{t+1}^k = \left(\frac{S_{t+1}^+}{S_t^+} \right) \lambda^+ R^+ - \left(\frac{S_{t+1}^-}{S_t^-} \right) \lambda^- R^- \quad (1)$$

$+$ index is used for the target currency and $-$ is used for the funding currency. $R^\pm = 1 + r^\pm$ is the gross interest rate of the target currency and of the funding currency. S_t is the exchange rate i.e. the amount of the foreign currency for one unit of arbitrageurs' currency. λ^+ and λ^- are the shares that the arbitrageurs use for the short and long positions in their portfolios. Since we assumed a log normal process for $\frac{S_{t+1}}{S_t}$, the wealth process follows a log normal process too. We assume the initial wealth of trader in this section is zero, while in leveraged carry trading the trader needs an initial wealth in order to cover the haircuts of short-selling/long-buying.

The first term is the profit due to the short position in the target currency and the second term is due to the long position in the funding currency.

We impose the following condition in the strategy of the arbitrageur:

$$\lambda^+ S^+ = \lambda^- S^- \quad (2)$$

This constraint means the amount of money arbitrageur borrows from the funding currency should be equal to the amount of money invested in the target currency.

If arbitrageurs are risk-neutral, they do care only about their expected payoffs and not the risk in the carry trade market. Hence, choosing the risk averse arbitrageurs allows us to build a trade off between payoff and risk. So, arbitrageurs maximize the following utility function: $U(W_{t+1}^k) = -e^{-\gamma_k W_{t+1}^k}$ where γ_k is the risk aversion parameter. This is a CARA utility function. Using this utility function with the normality assumption for the wealth process results in a mean-variance utility function.

4.3 Solving the model

The maximization problem is:

$$\max_{\lambda^+, \lambda^-} \mathbb{E}(W_{t+1}) - \frac{\gamma}{2} \text{Var}(W_{t+1})$$

s.t.

$$\lambda^+ S^+ = \lambda^- S^-$$

The maximization problem for λ^+ and λ^- under the constraint yields:

$$\lambda^+ = \frac{R^+\Gamma^+ - \left(\frac{S_t^+}{S_t^-}\right) R^-\Gamma^-}{\gamma_k \left[R^{+2}\Sigma_{S^+}^2 + R^{-2}\Sigma_{S^-}^2 \left(\frac{S_t^+}{S_t^-}\right)^2 - 2R^+R^- \left(\frac{S_t^+}{S_t^-}\right) (\Omega - \Gamma^+\Gamma^-) \right]} \quad (3)$$

$$\lambda^- = \frac{\left(\frac{S_t^-}{S_t^+}\right) R^+\Gamma^+ - R^-\Gamma^-}{\gamma_k \left[R^{+2}\Sigma_{S^+}^2 \left(\frac{S_t^-}{S_t^+}\right)^2 + R^{-2}\Sigma_{S^-}^2 - 2R^+R^- \left(\frac{S_t^-}{S_t^+}\right) (\Omega - \Gamma^+\Gamma^-) \right]} \quad (4)$$

where Γ is the expected value of depreciation (appreciation) of each currency and Σ_S^2 is its variance. Ω is a constant which is expressed in the appendix. The nominators of these fractions are the adjusted return to the exchange rate movements and the denominators are risk factors. These equations express that the amount of money in long position should increase when the interest rates differential goes up and should decrease whenever the risk of currency crash is high. The risk of currency, here, is measured by the volatility of the exchange rates, i.e., $\Sigma_{S^\pm}^2$. The relative risk aversion parameter is appeared in the denominator, which is consistent with intuition. The last term in the denominators is the covariance between the target and the funding currency and since the dynamics of both currencies are affected by the same Brownian process, this term appears with a negative sign in the risk term (denominator here) and if the two stochastic processes are not correlated this term vanishes.

According to these equations, an arbitrageur puts in the carry trade market if he expects a positive return. This result is similar to Jylh *et al* (2008).

Proposition 1 *Arbitrageur borrows in the funding currency and lends in the target currency, if and only if the expected returns from carry trade is strictly positive i.e., $R^+\Gamma^+ > \left(\frac{S_t^+}{S_t^-}\right) R^-\Gamma^-$.*

Arbitrageur borrows in the funding currency and lends in the target currency means $\lambda^\pm > 0$. The denominators of equations 3 and 4 are positive. Therefore the positions are positive if and only if the nominators are positive, from which we deduce the result.

4.4 The impact of the number of arbitrageurs

The same technique is used in order to maximize the profit of lender/borrower in the funding and target currencies. The dynamic of wealth for a funding

currency borrower is:

$$W_{t+1}^- = R^- \lambda_F^- \frac{S_t^-}{S_{t+1}^-}$$

The same form of utility function is applied for this traders. Therefore, the maximization problem is:

$$\max_{\lambda_F^-} \mathbb{E} \left(-e^{-\gamma^- W_{t+1}^-} \right)$$

With $\Gamma_F = e^{-\mu^{-2} + \sigma^{-2}}$, the maximization problem yields

$$\Rightarrow \lambda_F^- R^- = \frac{\Gamma_F}{\gamma^- \Sigma_{S^-}^2}$$

This latter equation helps us to simplify the calculations furthermore by substituting the $\lambda_F^- R^-$ with a constant. Next we verify the market clearing condition between the arbitrageurs and the funding currency supplier. Assuming K arbitrageurs and one aggregate supplier, the market clearing condition can be written as:

$$\sum_{k=1}^K \lambda_k^- = \lambda_F^-$$

Suppose that all arbitrageurs have the same risk aversion parameter, ($\frac{1}{\gamma_k} = \rho = \text{const}$), and with defining $\alpha = \lambda^- \gamma_k$ we get:

$$\alpha \rho K = \lambda_F^-$$

$$\lim_{K \rightarrow \infty} \alpha = 0$$

which means that the nominator of equation (3) should tend to zero. In other words, we have

$$\frac{S_t^+}{S_t^+} R^+ \Gamma^+ = R^- \Gamma^-$$

This condition is the UIP condition. However the process of the exchange rate is assumed to be exogenous and therefore it can not be affected by the number of traders in the market. If the number of traders goes to infinity, their positions goes to infinite small because of lenders' limited source and their profit should tend to zero as well. A development of the model can take place by integrating the number of traders in the exchange rate process.

4.5 Sharpe Ratio

In this section, the goal is to compute the Sharpe ratio and see how it varies with different parameters of the model. The Sharpe ratio is a measure of the excess return per unit of risk. It is defined as:

$$SR = \frac{\mathbb{E} \left[\lambda^+ \frac{S_{t+1}^+}{S_t^+} R^+ - \lambda^- \frac{S_{t+1}^-}{S_t^-} R^- \right]}{\left[\text{Var} \left(\lambda^+ \frac{S_{t+1}^+}{S_t^+} R^+ - \lambda^- \frac{S_{t+1}^-}{S_t^-} R^- \right) \right]^{\frac{1}{2}}}$$

With some further manipulations, we have

$$SR = \frac{\left(\frac{S_t^-}{S_t^+} \right) R^+ \Gamma^+ - R^- \Gamma^-}{\left((R^+)^2 \left(\frac{S_t^-}{S_t^+} \right)^2 \Sigma_+^2 + (R^-)^2 \Sigma_-^2 - 2(R^+ R^-) \left(\frac{S_t^-}{S_t^+} \right) [\Omega - \Gamma^+ \Gamma^-] \right)^{\frac{1}{2}}}$$

Using equation (3) we are able to rewrite the Sharpe ratio:

$$(SR)^2 = \gamma_k \lambda^- \left[\left(\frac{S_t^-}{S_t^+} \right) R^+ \Gamma^+ - R^- \Gamma^- \right] \quad (5)$$

Hence the square of Sharpe ratio is a linear function of the arbitrageurs' long position (It also can be expressed in term of long position). Also when $k \rightarrow \infty$, $\left(\frac{S_t^-}{S_t^+} \right) R^+ \Gamma^+ = R^- \Gamma^-$ and consequently the Sharpe ratio tends to zero. Null Sharpe ratio comes from the fact that a trader's position in the presence of infinite number of traders goes to zero.

Lemma 1 *When the number of the traders tends to infinity, the arbitrage opportunity vanishes and the Sharpe ratio tend to zero.*

Since we see big profit on the carry trade market in normal periods, the above lemma means the carry trade market is characterized by a non-competitive structure. However this non-competitive structure is not due to the entering barriers, excessive collateral, asymmetric information or higher trading costs which are the general causes of many other non-competitive markets, because trading on Forex is possible for everybody with an access to internet. Transaction costs in Forex are very low and information on exchange rates and interest rates are instantly available for every trader.

This lemma means the bigger is the number of arbitrageurs, the more competitive is the carry trade market, and the less reasonable is to choose a riskier action in order to get a bigger profit. Since the lenders have limited resources, if the number of traders goes to infinity, each trader would have an infinite small share to borrow and therefore its profit due to applying carry trade strategies would be very close to zero. The profit must disappear whatever the level of risk is. In other words, when the number of trader increases, the market becomes more competitive profit must vanish whatever the level of risk is.

4.6 Concavity of the Sharpe ratio

As our model is based on mean-variance optimization, the use of Sharpe ratio is meaningful. According to the mean-variance and the CAPM theories, portfolios with the highest Sharpe ratio are mean-variance efficient. Moreover, what we find in the equilibrium is a mean-variance efficient portfolio. Therefore to choose this portfolio or in other words, to maximise the arbitrageurs utility function, we should maximize the Sharpe ratio.

In this part, we show that the Sharpe ratio has some maximum relative to the interest rates differentials. Otherwise, if the interest rate differentials are bigger than some thresholds, the Sharpe ratio decreases. This is maybe due to the crash risk of the currencies with the high interest rate. Thus, we want to confirm the fact shown in the figure (6). For this purpose, we have to calculate the first and the second derivatives of the Sharpe ratio with respect to the interest rate differentials. From the market clearing condition we have:

$$K\lambda^- = \lambda_F^-$$

multiplying by R^-

$$\lambda^- R^- = \frac{\Gamma_F}{\gamma^- K \Sigma_-^2}$$

Substituting this in the Sharpe ratio relation (Eq. 5):

$$SR^2 = \frac{\gamma_k}{\gamma^-} \frac{\Gamma_F}{K \Sigma_-^2} \left[\left(\frac{S_t^-}{S_t^+} \right) \left(\frac{R^+}{R^-} \right) \Gamma^+ - \Gamma^- \right] \quad (6)$$

Let's rewrite the Sharpe ratio equation by using the following notation: $R = \frac{R^+}{R^-}$ and $S_t = \frac{S_t^-}{S_t^+}$.

$$SR^2 = \frac{(S_t R \Gamma^+ - \Gamma^-)^2}{R^2 S_t^2 \Sigma_+^2 + \Sigma_-^2 - 2RS_t[\Omega - \Gamma^+ \Gamma^-]}$$

Now we are able to use this formula to calculate the maximum of the Sharpe ratio.

$$2(SR) \frac{\partial(SR)}{\partial R} = 0$$

This gives an equation of 4th degree. To see the concavity of the Sharpe ratio, we derive equation (6). The first derivative is

$$2(SR) \frac{\partial(SR)}{\partial R} = \frac{\gamma}{\gamma^-} \frac{\Gamma_F}{K \Sigma_{S^-}^2} \left(\frac{S_t^-}{S_t^+} \right) \Gamma^+$$

The *rhs* of the above equation is constant and therefore the second derivative can be calculated as follow:

$$\frac{\partial^2(SR)}{\partial R^2} = -\frac{1}{(SR)} \left(\frac{\partial(SR)}{\partial R} \right)^2 < 0$$

This second derivative is negative if and only if

$$(SR)_k > 0 \Leftrightarrow \frac{S_t^-}{S_t^+} R^+ \Gamma^+ > R^- \Gamma^-$$

which means that the interest rate of the target currency adjusted to the exchange rate movement should be higher than the interest rate of the funding currency. Using Proposition (1), we know that this is always true for an arbitrageur who wants enter into this market. Thus the second derivative is negative and the Sharpe ratio has a concave form respect to the interest rate differentials. We can consider any portfolio with smaller Sharpe ratio as non-efficient and therefore it is not optimal for the investors to choose it.

Proposition 2 *In the carry trade market, the Sharpe ratio has a concave form with respect to the interest rates differentials.*

Figure (6) shows the Sharpe ratio versus interest rate differentials which is calculated for different currencies. As we showed in the theoretical part, the Sharpe ratio has a concave form. Here the maximum takes place for the Hungarian forint. The figure shows that the excess return due to the high interest rate differentials is compensated by the high risk of currency crash for Brazilian Real and Turkish Lira.

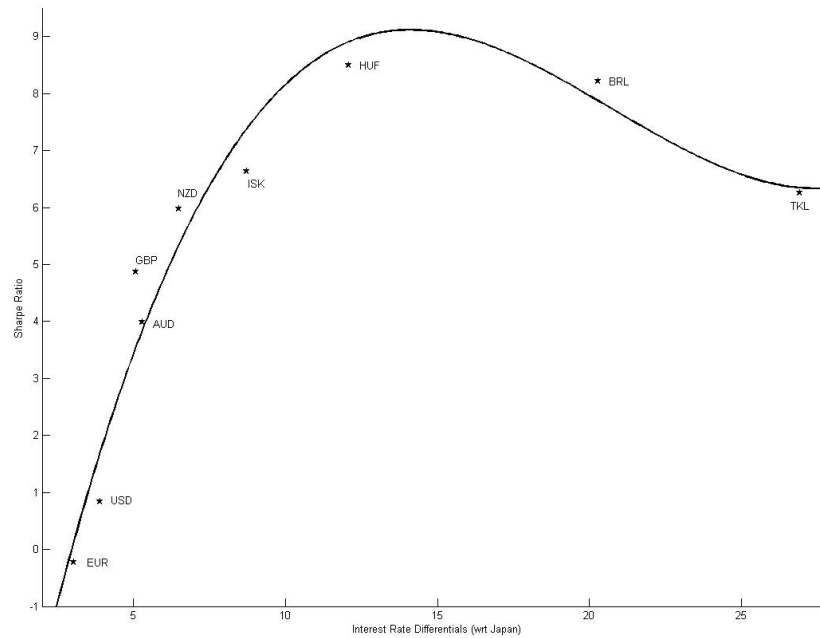
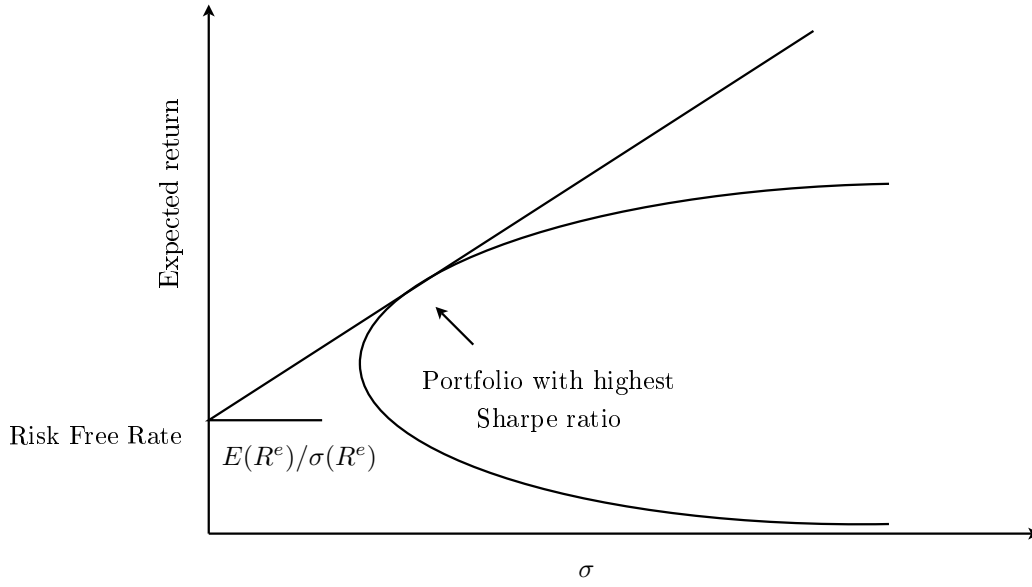


Figure 6: Sharpe ratio vs interest rate differentials 1996-2008.



The slope of tangency portfolio gives the maximum Sharpe ratio. This slope has an increasing form from a negative range toward its maximum which reaches at the tangency portfolio. Therefore, as the above figure shows for a mean-variance efficient portfolio, the maximum of Sharpe ratio takes place at the tangency portfolio. This happens on the figure 6 for the Hungarian forint. Therefore it is the tangency portfolio. Figure 6 suggests also that the log-normality for the wealth process can be a suitable hypothesis, since this figure confirms the concave form of the Sharpe ratio in which the model predicts.

4.7 Leveraged carry trade

Carry trade activities are a part of spot Forex market. Brokers debit or credit traders' account according to overnight interest rate differential between two currencies. In this section we build a carry trade strategy using an amount of leverage available to traders. In the Forex market, the leverage ratio is generally very high and this makes carry trade an attractive strategy for investment.

However trading in the market requires an initial capital. When a trader buys an asset, she can use it as collateral and she can buy against this collateral. Nevertheless she can not borrow for the total value of the asset. Otherwise she has to put some of her initial capital. This initial capital which is the difference between the asset price and amount of borrowed money is called the margin or haircut. The inverse of the margin rate is the leverage ratio. Short selling as well as buying long requires an initial capital. Margins or haircuts are set by the Forex brokers and they depend on their beliefs of future states of assets (for a trader they are exogenous).

We assume the process of wealth is as before in equation (1). However we change the constraint (2). Assume the initial wealth of the trader is W_0 . m^\pm are the margins

for selling short and buying long. When a trader takes a long position, her account will be credited at overnight rates by brokers. However her account is debited when she takes a short position. Therefore the wealth process of equation(1) still holds. The budget constraint is:

$$W_0 \geq m^- \lambda^- - m^+ \lambda^+ \quad (7)$$

Since the return follows a log-normal process, we assume directly the mean-variance utility function. The program of maximisation is as follow:

$$\begin{aligned} & \mathbb{E}(W_{t+1}) - \frac{\gamma}{2} \text{Var}(W_{t+1}) \\ & \text{s.t constraint (7)} \end{aligned}$$

Solving this maximisation problem we get the following expressions for the short and long position of a trader

$$\lambda^+ = \frac{\frac{R^+}{m^+} \Gamma^+ - \frac{R^-}{m^-} \Gamma^- + \frac{\gamma W_0}{m^-} \left(\frac{R^+ R^-}{m^+} (\Omega - \Gamma^+ \Gamma^-) - \frac{(R^-)^2}{m^-} \Sigma_-^2 \right)}{\gamma m^+ \left[\frac{(R^+)^2}{(m^+)^2} \Sigma_+^2 + \frac{(R^-)^2}{(m^-)^2} \Sigma_-^2 - \frac{2R^+ R^- (\Omega - \Gamma^+ \Gamma^-)}{m^+ m^-} \right]} \quad (8)$$

and

$$\lambda^- = \frac{\frac{R^+}{m^+} \Gamma^+ - \frac{R^-}{m^-} \Gamma^- + \frac{\gamma W_0}{m^+} \left(\frac{(R^+)^2}{m^+} \Sigma_+^2 - \frac{R^+ R^-}{m^-} (\Omega - \Gamma^+ \Gamma^-) \right)}{\gamma m^- \left[\frac{(R^+)^2}{(m^+)^2} \Sigma_+^2 + \frac{(R^-)^2}{(m^-)^2} \Sigma_-^2 - \frac{2R^+ R^- (\Omega - \Gamma^+ \Gamma^-)}{m^+ m^-} \right]} \quad (9)$$

These equations are very hard to interpret. To have more insight through these equations we rewrite them in a simpler form: let

$$\pi_{t+1} = \frac{R^+}{m^+} \left(\frac{S_{t+1}^+}{S_t^+} \right) - \frac{R^-}{m^-} \left(\frac{S_{t+1}^-}{S_t^-} \right)$$

Then the positions are:

$$\lambda^+ = \frac{\mathbb{E}_t(\pi_{t+1}) + \gamma W_0 \text{Cov}_t \left(\pi_{t+1}, \frac{R^-}{m^-} \left(\frac{S_{t+1}^-}{S_t^-} \right) \right)}{\gamma \text{Var}_t(\pi_{t+1})} \quad (10)$$

$$\lambda^- = \frac{\mathbb{E}_t(\pi_{t+1}) + \gamma W_0 \text{Cov}_t \left(\pi_{t+1}, \frac{R^+}{m^+} \left(\frac{S_{t+1}^+}{S_t^+} \right) \right)}{\gamma \text{Var}_t(\pi_{t+1})} \quad (11)$$

$R^+ \frac{S_{t+1}^+}{S_t^+} - R^- \frac{S_{t+1}^-}{S_t^-}$ is the risk premium for short selling and buying long one unit of currency. $L^+ = \frac{1}{m^+}$ and $L^- = \frac{1}{m^-}$ are the leverage ratios for the short and long positions. Therefore π_{t+1} is the *leveraged risk premium* of investing one unit in each currency, short one currency and long another one.

The denominators of the positions are the variance of the leveraged risk premium multiplied by the coefficient of the risk aversion. The nominators are the expected

value of leveraged risk premium plus a term of covariance. For example if the leveraged risk premium covary with funding currency positively, equation (10) suggests that the trader should reinforce her short-selling position. In other words, according to the equation (8) if the two currencies covary positively with each other and the variance of the funding currency is small, then the risk in the market decreases and this encourages the trader to take a bigger position. The same interpretation can be given to the equation (11).

The positions should be positive. Since the denominators are positive only the nominators should be positive. This is similar to the proposition 1. However here the nominators contains a term of covariance and their positivity are not straight forward.

Lemma 2 *The short position is decreasing with respect to m^+ if and only if the leveraged volatility of funding currency is bigger than the leveraged volatility of the target currency adjusted by the interest rates. In other term:*

$$\left(\frac{R^-}{m^-}\right)^2 \Sigma_-^2 - \left(\frac{R^+}{m^+}\right)^2 \Sigma_+^2 \geq 0$$

The condition for the long position with respect to m^- is the inverse of above condition.

While we assume the positions are positive, there still exist one term which its positivity is not guarantied in the derivation of λ^+ with respect to m^+ . This term is the leverage volatility (or its square, variance) adjusted by the interest rate, i.e, the mathematical expression in the proposition. This condition is an enough condition, however it is not necessary for lemma to hold.

Proposition 3 *If the condition in the lemma 2 holds, then the carry trade return is also decreasing with respect to m^+ and with the same condition it will be increasing with respect to m^- .*

Since at the optimal point the constraint is binding, we can rewrite the $\frac{\partial W_{t+1}}{\partial m^+}$ by using the constraint, only in terms of $\frac{\partial \lambda^+}{\partial m^+}$. Therefore we have:

$$\frac{\partial W_{t+1}}{\partial m^+} = \frac{\partial \lambda^+}{\partial m^+}(\pi_{t+1}) - \frac{\lambda^+}{m^-} R^- \left(\frac{S_{t+1}^-}{S_t^-} \right)$$

Since we expect the risk premium to be positive, if the condition in the lemma 2 holds, then the return (and not the utility function) would be decreasing with respect to the funding currency margin, i.e, m^+ . Intuitively, this is because m^+ is already very high comparing to m^- adjusted by the interest rates and the volatilities.

5 Conclusion

We tried to construct a portfolio for a trader in a third party country. The traders' strategy is unhedged in this research. In other words, traders do not immune themselves against target currency depreciation or funding currency appreciation and by buying some forward contracts.

Using a Panel VAR, we show that the profit due to the carry trade is very sensitive to the exchange rate movements and interest rate differentials profit. Although the exchange rate profit is negligible and even negative, it affects the total profit so much. Skewness and Kurtosis are depicted for all currency pairs. All currencies are negatively skewed relative to the Japanese yen and their kurtosis is positive. This is the case even for the currencies (as euro and pound) that normally are not used for carry trade. Negative skewness means that these currencies are exposed to a crash risk. Average skewness and kurtosis is plotted against interest rates differentials. The figures depict that higher interest rate currencies have more negative skewness and more positive kurtosis. This means higher return currencies are riskier. The 9 currencies can be divided into three sub groups according to their return and their exchange rate risk exposure. The exchange rates of currencies in each group have a co-movement with each other.

Finally excess profit and Sharpe ratio are computed. Sharpe ratio is a measure of profitability. The carry trade market offers a very high Sharpe ratio comparing to the other markets. First Sharpe ratio increases for the high return currencies but after a while it decreases. The concavity of the Sharpe ratio is shown by the model too. Thus choosing between currencies to invest depends on how much traders are risk reversal.

In the theoretical part, the model shows that the arbitrageur put in the market, whenever they expect that the UIP does not hold. Next, we showed that when the number of trader goes to infinity, the carry trade opportunity vanishes and it looks like a competitive market. In term of the Sharpe ratio, this is interpreted as non-profitability and therefore the Sharpe ratio tends to zero. The important contribution of the model is to show that the Sharpe ratio is concave.

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APPENDIX

5.1 Interest rates evolution

The following figures show different interest rates classified by target and funding currency.

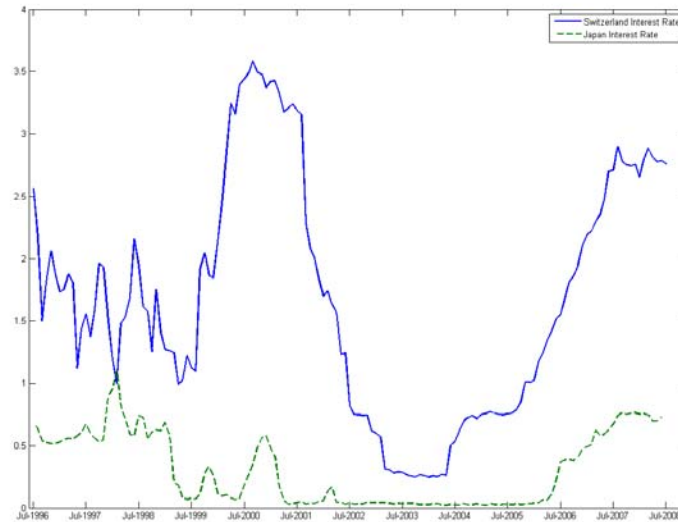


Figure 7: Funding Currencies.

Japanese Yen and Swiss Franc are supposed to be the funding currencies because they have the lowest interest rate among other currencies of our sample

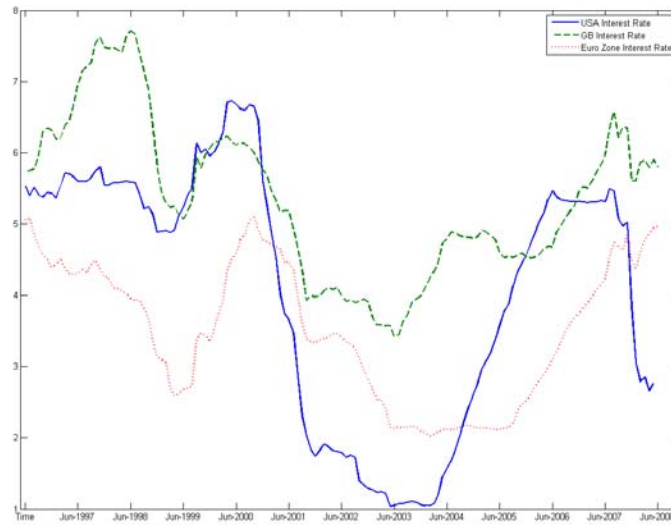


Figure 8: Occasionally target currencies.

US dollar is supposed to be an occasionally target currency. In fact this was true until recently. Now the US interest rate is practically zero and it can be regarded as a new funding currency. The interest rate in the Euro zone has almost similar pattern, with the difference that its volatility is lower than the US interest rate.

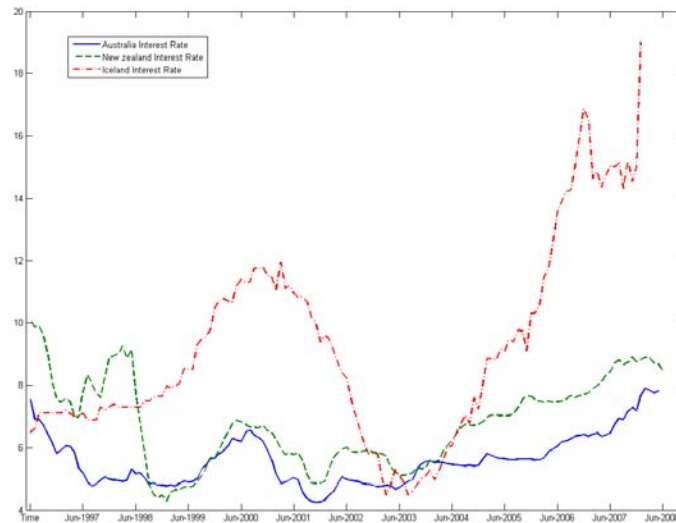


Figure 9: Target Currencies with low IR.

This group of currencies are the popular destination for the carry traders, since these currencies have relatively stable economies (except Iceland recently) relative to the developing countries and they offer high interest rates.

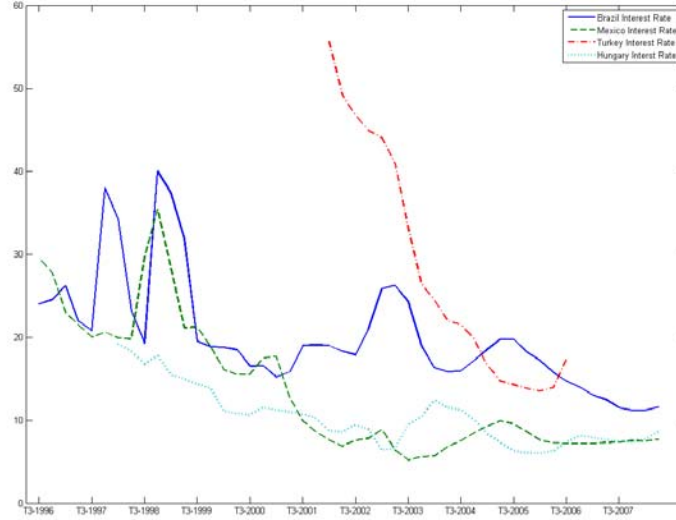


Figure 10: Target Currencies with high IR.

The last group of currencies offer very high interest rates but they have a high volatility both in the interest rate and in the exchange rate and therefore the carry trade expose to a higher return crash risk.

The maximization problem

In this part, the notations and calculations are shown in detail. Using the dynamic of the profit, we can write:

$$\mathbb{E} \left(-e^{-\gamma_k W_{t+1}^k} \right) = -\mathbb{E} \exp \left[-\gamma_k \lambda^+ \frac{S_{t+1}^+}{S_t^+} R^+ + \gamma_k \lambda^- \frac{S_{t+1}^-}{S_t^-} R^- \right]$$

We use the following notation for the mean and variance of the exchange rate:

$$\begin{aligned} \mathbb{E} \left(\frac{S_{t+1}}{S_t} \right) &= e^{\left(\mu - \frac{\sigma^2}{2} \right)} \mathbb{E} e^{\sigma(W_{t+1} - W_t)} \\ &= e^{\left(\mu - \frac{\sigma^2}{2} \right)} e^{\frac{\sigma^2}{2}} = e^\mu = \Gamma \end{aligned}$$

and for the variance

$$\begin{aligned}
\mathbf{VAR}\left(\frac{S_{t+1}}{S_t}\right) &= \mathbf{VAR}\left(e^{\left(\mu - \frac{\sigma^2}{2}\right) + \sigma(W_{t+1} - W_t)}\right) \\
&= e^{(2\mu - \sigma^2)} \left[\mathbb{E}e^{2\sigma(W_{t+1} - W_t)} - \left(\mathbb{E}e^{\sigma(W_{t+1} - W_t)}\right)^2 \right] \\
&= e^{(2\mu - \sigma^2)} \left[e^{2\sigma^2} - e^{\sigma^2} \right] = e^{2\mu} (e^{\sigma^2} - 1) \\
&= \Sigma_S^2
\end{aligned}$$

then, we can write

$$\mathbb{E}\left[-\gamma_k \lambda^+ \frac{S_{t+1}^-}{S_t^-} R^+ + \gamma_k \lambda^+ \frac{S_{t+1}^-}{S_t^-} R^-\right] = -\gamma_k \lambda^+ R^+ \Gamma^+ + \gamma_k \lambda^- R^- \Gamma^-$$

and

$$\mathbf{VAR}\left[-\gamma_k \lambda^+ \frac{S_{t+1}^+}{S_t^+} R^+ + \gamma_k \lambda^- \frac{S_{t+1}^-}{S_t^-} R^-\right] = \gamma_k^2 \left[\left(\lambda^+ R^+\right)^2 \Sigma_{S^+}^2 + \left(\lambda^- R^-\right)^2 \Sigma_{S^-}^2 - 2 \left(\lambda^+ \lambda^-\right) \left(R^+ R^-\right) \left[\Omega - \Gamma^+ \Gamma^-\right] \right]$$

with

$$\Omega = e^{(\mu^+ + \mu^- + \sigma^+ \sigma^-)}$$

and the expected value of the utility function is:

$$\max_{\lambda^+, \lambda^-} -\exp\left[-\gamma_k \left(\lambda^+ R^+ \Gamma^+ - \lambda^- R^- \Gamma^-\right) + \frac{\gamma_k^2}{2} \left[\left(\lambda^+ R^+\right)^2 \Sigma_{S^+}^2 + \left(\lambda^- R^-\right)^2 \Sigma_{S^-}^2 - 2 \left(\lambda^+ \lambda^-\right) \left(R^+ R^-\right) \left[\Omega - \Gamma^+ \Gamma^-\right] \right] \right]$$

Skewness and Kurtosis

In the following figures skewness and kurtosis of all currencies versus Japanese yen are shown.

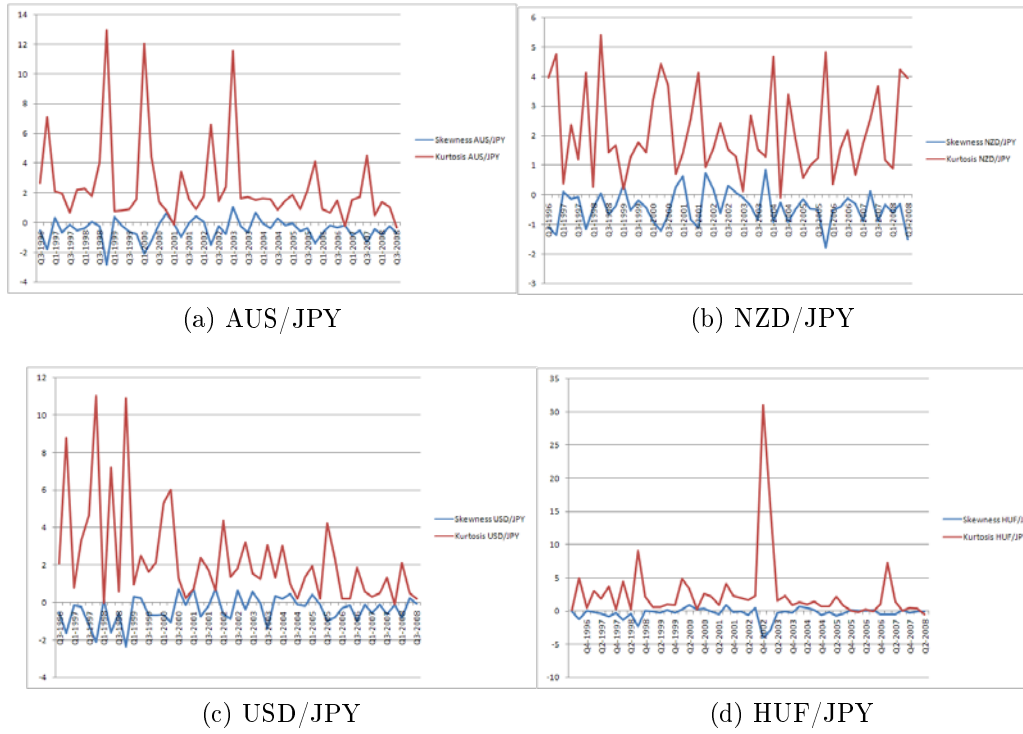
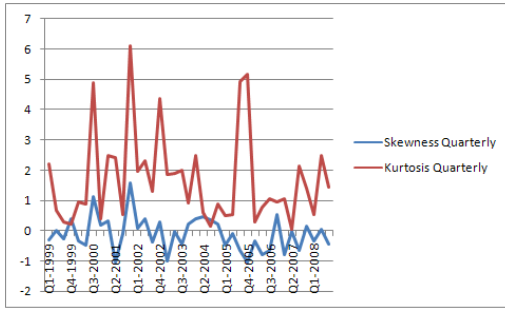
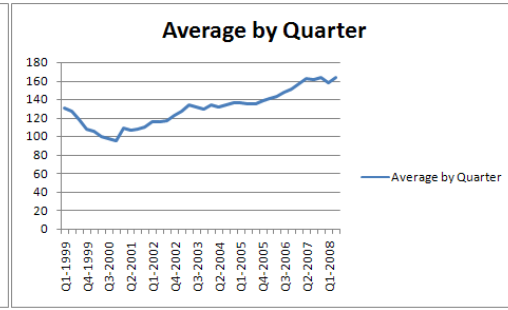


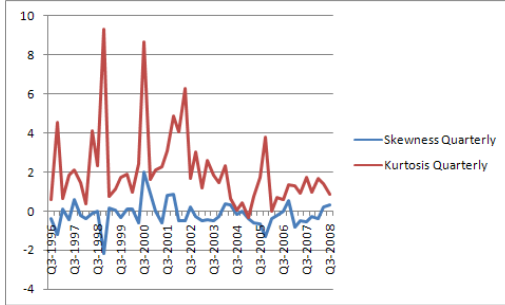
Figure 11: Skewness and Kurtosis for some target currencies



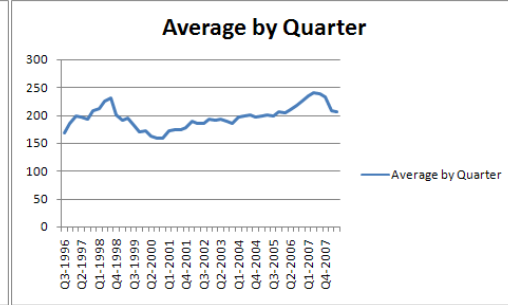
(a) Skewness and Kurtosis for euro quarterly



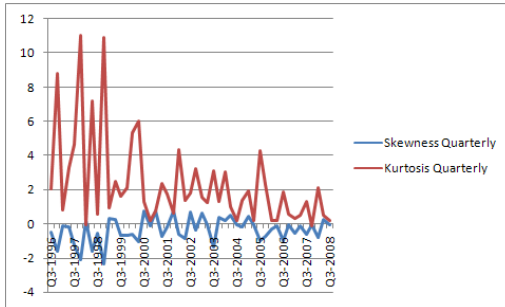
(b) EUR/JPY quarterly



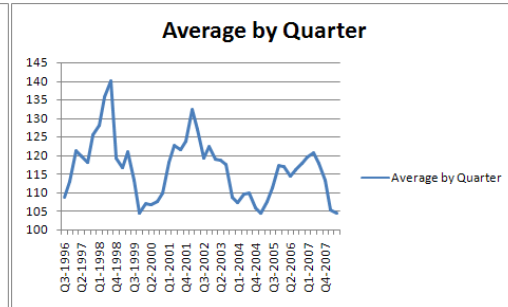
(c) Skewness and Kurtosis for pound sterling quarterly



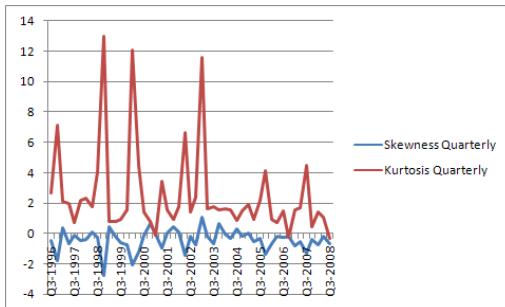
(d) GBP/JPY quarterly



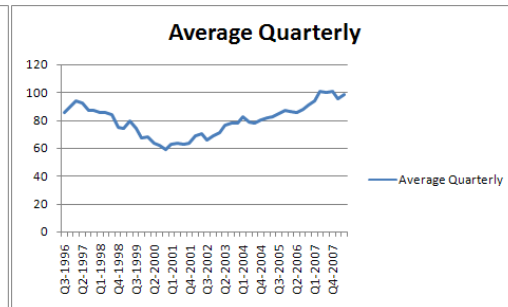
(e) Skewness and Kurtosis for USD quarterly



(f) USD/JPY quarterly

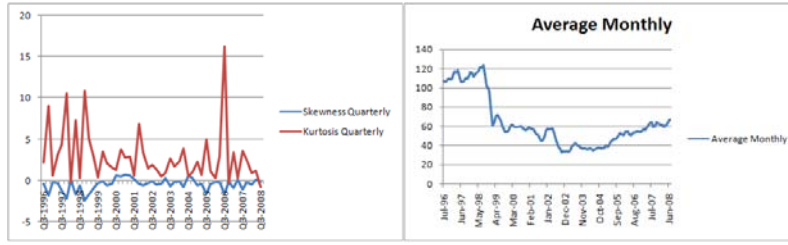


(g) Skewness and Kurtosis for AUD quarterly

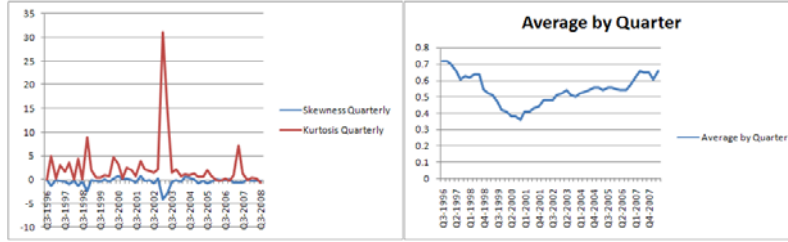


(h) AUD/JPY quarterly

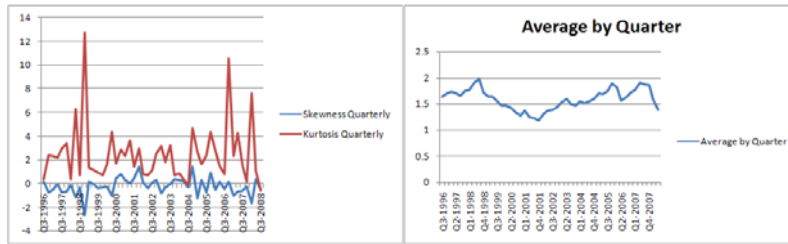
Figure 12: Skewness and Kurtosis quarterly-Exchange rate vs Japanese yen



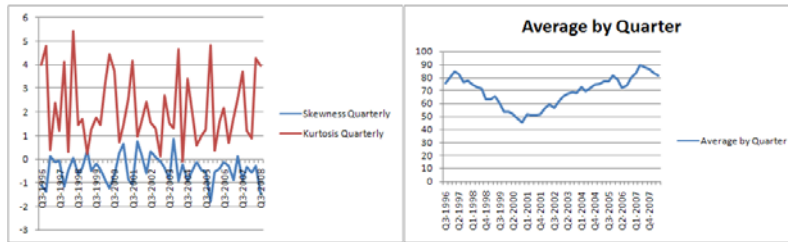
(a) Skewness and Kurtosis for BRL quarterly (b) BRL/JPY quarterly



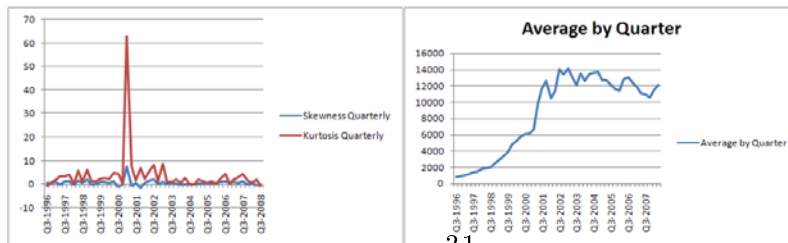
(c) Skewness and Kurtosis for HUF quarterly (d) HUF/JPY quarterly



(e) Skewness and Kurtosis for ISK quarterly (f) ISK/JPY quarterly



(g) Skewness and Kurtosis for NZD quarterly (h) NZD/JPY quarterly



(i) Skewness and Kurtosis for TKL quarterly (j) JPY/TKL quarterly

Figure 13: Skewness and Kurtosis quarterly-Exchange rate vs Japanese yen-high return currencies